Problem set 1

This problem set is due at 12 noon on Thursday 24 January. Late problem sets are not accepted— no exceptions. See the syllabus for details.

Please answer all of the following questions. Provide the arguments and/or derivations leading to your answers, but do not write lengthy essays. You will need some calculus. Whenever you get stuck, first try to provide an answer based on some more informal intuition. If you get really stuck, consult your class teacher or me.

1. Consider a small sector with a single (or representative) firm. The firm maximizes its profits, taking wages in the labour market as given. The firm's profits are

   $$\pi(w, N) = R(N) - wN$$

   if it employs N workers at a wage w, where, for some $\delta > 0$ and $\gamma > 1$,

   $$R(N) = \frac{\delta\gamma}{\gamma - 1}\left(\frac{N}{\delta}\right)^{(\gamma-1)/\gamma}$$

   are the firm's revenues as a function of its labour input.

   (a) Verify that $R(0) = 0$ and that $R'(N) > 0$ and $R''(N) < 0$ for all $N > 0$. How does $R'(N)$ behave if you let N decrease to 0? And what happens to $R'(N)$ if you let N grow very large? What is the economic significance of all this?

   (b) What is the firm's demand for labour $N(w)$ as a function of the wage w?

   (c) What is the wage-elasticity of labour demand?

   (d) Explain: "An increase in δ corresponds to an iso-elastic increase in labour demand".

2. Now suppose that all workers inelastically supply one unit of labour (say a 40 hour work week) to the economy. Recall that the sector we are studying is small and suppose that workers can earn a wage $w_0 > 0$ elsewhere in the economy. Denote sectoral labour demand $N(w_0)$ at the wage w_0 by N_0. Workers derive utility $u(w)$ from consuming their income w. Suppose that

   $$u(w) = w^\alpha$$

   for some $\alpha > 0$. Workers employed in the sector at a wage w have utility $u(w)$; workers employed elsewhere enjoy utility $u(w_0)$.

(a) Verify that $u'(w) > 0$ for all $w > 0$. What is the economic significance of this? For what values of the parameter α are workers respectively risk-averse, risk-neutral and risk-loving? In the remainder, assume that workers are either risk-averse or risk-neutral.

(b) One measure of the degree of risk-aversion is the "Arrow-Pratt measure of relative risk-aversion" $-wu''(w)/u'(w)$. The larger this measure, the more risk-averse the worker is. What is the measure of relative risk-aversion in our case? Does it depend on the income level w? How does the degree of risk-aversion vary with the parameter α?

(c) How does labour supply to the sector vary with the sectoral wage?

(d) What are wages and employment in the sector if wages are set competitively?

3. Now suppose instead that $M > N_0$ workers organize in a trade union that is able to control all labour supply to the sector. The trade union acts as a monopolistic supplier of labour to the sector and sets wages as to maximize a utilitarian union objective function. The firm then chooses employment taking the wage demanded by the union as given.

(a) Give the utilitarian union utility function for our case. Provide two interpretations, one from the perspective of the union members before the union jobs are distributed among these members and another from the perspective of the union members after the union jobs are distributed among them.

(b) Recalling that the sector is small, what is the minimum level of union utility that the union can guarantee without even exerting any market power in the unionized sector? What does the iso-utility curve corresponding to this union utility level look like? What are the main properties of the iso-utility curves corresponding to higher utility levels? Illustrate your answer in a diagram with N on the horizontal and w on the vertical axis.

(c) Would the union ever settle for a wage below w_0, for example in return for additional employment?

(d) Provide an (implicit) expression for wages and employment in the union sector in terms of the parameters α, δ and γ in the "monopoly-union" situation sketched above. How do monopoly-union wages and employment compare to the competitive case?

(e) Consider an iso-elastic increase in labour demand. What happens to employment and wages in the competitive case? And in the monopoly-union case? Provide some economic intuition.

(f) Answer the same question for an increase in workers' risk aversion.

(g) Answer the same question for an increase in w_0.

(h) How would the monopoly-union wage change if we vary δ and γ such that labour demand becomes more elastic without affecting the competitive outcome (if you like: a rotation of the labour demand curve around the competitive equilibrium)? What is the effect of such a change on wages in the monopoly-union case? What can you say about employment effects? Provide some economic intuition.

[*Hint: Denote the competitive equilibrium by* $(\bar{w}, \bar{N})$*. Note that* $\bar{N} = N(\bar{w})$ *and that the labour demand function* N *depends on* δ *and* γ*. So, if we change* γ*, we typically change* $N(\bar{w})$ *and therefore* $\bar{N}$*, for given* $\bar{w}$*. We can however simultaneously change* δ *to ensure that* $N(\bar{w})$ *remains constant.*]

4. We continue with some questions on efficiency.

   (a) What are the main properties of the iso-profit curves of the firm of the previous questions? In particular, discuss the relation with the labour demand curve. Illustrate your answer in a diagram with N on the horizontal and w on the vertical axis.

   (b) Argue that the monopoly-union outcome is (Pareto) inefficient.

   (c) Show that the competitive outcome is on one end of the contract curve.

   (d) Take an educated guess at the shape of the contract curve. Illustrate your guess in a diagram with N on the horizontal and w on the vertical axis.

5. Read the article "'How rail privatisation gave unions more power" in The Independent of 9 January (follow the link on the website in the course material section). *Briefly* comment on Lew Adams' "privately harboured politically incorrect thoughts" that The Independendent makes public here. You may also have some thoughts on the relative success of his union Aslef.