

## Solutions for problem set 4

1.

- (a) The worker is risk-averse and prefers a risk-free (fixed) wage over a risky (output-dependent) wage with mean equal to the risk-free wage. The firm is risk-neutral and, as there are no moral hazard problems, is indifferent between paying a more or less risky wage (with identical mean). Thus, the firm will offer a fixed wage (risk-free) contract. If it would offer a risky contract, it would have to compensate the worker for the additional risk with a higher expected wage and the firm's expected profits would be lower.

In order to be able to hire some (shirking) worker, the farm must give her/him more utility than the utility  $\sqrt{70}$  he/she derives from the outside option of a certain £70. We already know the farm will offer a fixed wage, so that this boils down to the restriction that the firm should offer at least £70. If the farm offers a wage above £70 the worker would be better off, but as it would also increase the farm's costs without a change in output. Thus, the profit-maximising firm chooses the lowest wage that satisfies the worker's participation constraint, £70.

As the worker receives a fixed wage, and the farm an uncertain profit (uncertain output minus fixed wage) it is the farm that bears all risk of the contract.

- (b) As in 1.a, if the farm is able to contract on the level of effort and enforce the contract (as assumed in this part of the exercise), it will find it optimal to offer a fixed-wage contract. A farm wanting to hire non-shirking workers must pay them (net of effort cost) at least their outside option. The fixed wage offered must therefore satisfy:

$$w - 10 \geq 70.$$

A worker would therefore accept to work and to provide the required effort  $e = 1$ , for any wage of at least £80. Profit-maximisation leads the farm to offer the lowest wage £80 feasible with the worker's participation constraint (same reasoning than in point 1.a).

- (c) A farm able to enforce the level of effort  $e$ , will propose the contract that maximises expected profits. So let us compare expected profits that on average

arise from  $(w, e) = (70, 0)$  and from  $(w, e) = (80, 1)$  :

$$\begin{aligned}\pi_0 &= \mathbb{E}[y - w|e = 0] = \mathbb{E}[y|e = 0] - \mathbb{E}[w|e = 0] \\ &= 70 - 70 = 0,\end{aligned}$$

and

$$\begin{aligned}\pi_1 &= \mathbb{E}[y - w|e = 1] = \mathbb{E}[y|e = 1] - \mathbb{E}[w|e = 1] \\ &= 90 - 80 = 10.\end{aligned}$$

As  $\pi_1 > \pi_0$ , the farm will offer the  $(w, e) = (80, 1)$  contract. The notation  $\pi_j$  denotes the expected profit given an effort level of  $j$ :  $\pi_j = \mathbb{E}[\pi|e = j]$ .

- (d) We have just shown that  $\pi_1 > \pi_0$ , hence, the firm is better off under the high wage, high effort contract. The worker compares the utility level achieved under both contracts:  $u(70 - 0) = \sqrt{70}$  and  $u(80 - 10) = \sqrt{80 - 10}$ . As both utility levels are the same, we conclude that the worker is indifferent between both situations and that the contract  $(w, e) = (80, 1)$  is more efficient than  $(w, e) = (70, 0)$ .

For Pareto-efficiency, we have to check that there is no third contract that makes one of both agents even better off without hurting the other. The optimal contract  $(w, e) = (80, 1)$  was derived as the contract that maximises profits under the worker participation (utility) constraint. So, profits cannot be increased without decreasing worker utility and the contract is Pareto-efficient (check microeconomics text book for this argument).

2.

- (a) When the wage is fixed at  $w$  independently of the level of output, there are no benefits to the worker from exerting effort (working). As there are costs of working (£10), the worker will shirk ( $e = 0$ ).

Knowing that the worker will shirk under any fixed-wage contract, the firm will choose a wage level of £70, just as in exercise 1a.

- (b) The contract is acceptable to the worker if it gives him an expected utility of at least the outside utility  $\sqrt{70}$ . The expected levels of production (and, in this case, the expected wages) are

$$\bar{y}_0 = \frac{1}{4}100 + \frac{3}{4}60 = 70$$

when  $e = 0$ , and

$$\bar{y}_1 = \frac{3}{4}100 + \frac{1}{4}60 = 90$$

when  $e = 1$ . In order to see whether the worker accepts the contract, we have to compare the *expected utility levels* achieved under the alternative contracts.<sup>1</sup> The expected utility for  $e = 0$  is

$$\bar{u}_0 = \frac{1}{4}\sqrt{100} + \frac{3}{4}\sqrt{60} = 8.3095,$$

whereas for  $e = 1$ ,

$$\bar{u}_1 = \frac{3}{4}\sqrt{100 - 10} + \frac{1}{4}\sqrt{60 - 10} = 8.8829.$$

Briefly, as  $\bar{u}_1 > \sqrt{70} > \bar{u}_0$ , we can reach two conclusions.<sup>2</sup> First, as  $\bar{u}_1 > \bar{u}_0$ , the worker will provide the effort level  $e = 1$ . Second, as  $\bar{u}_1 > \sqrt{70}$ , the worker will supply her/his labour to the farm at wage  $w = y$ .

- (c) The second contract is more efficient because profits are the same and the worker is better off ( $\bar{u}_1$  is greater than  $\sqrt{70}$ ).
- (d) In the situation where  $w$  is constant (and thus certain), the worker will bear no risk, whereas the farm bears the whole risk as its profits  $y - w$  are uncertain. The worker shirks because the contract provides no incentives to work.

In the case where  $w = y$  the worker bears the whole risk. The worker works because the contract provides large rewards for working (i.e. increasing output). The firm will have 0 profits whatever happens.

It has been shown (see 1.c) that the farm could have a positive (and therefore higher) profit if effort level  $e = 1$  would be enforceable. So, ideally (in absence of moral hazard problems) the firm would like the worker to work. We will now argue that, even under asymmetric information and moral hazard, the farm can design a contract that gives it some positive profit, the worker an incentive to choose  $e = 1$  and sufficient utility to participate.

The key is that in the case that  $w = y$  neither the incentive-compatibility and the participation constraints are binding ( $\bar{u}_1 > \bar{u}_0$  and  $\bar{u}_1 > \sqrt{70}$ ). Thus, starting from  $w = y$  the firm can increase expected profits above 0 without violating the participation constraint by offering a slightly less generous contract to the worker. Note that the firm prefers this contract over the fixed wage contract.

Furthermore, the firm can improve efficiency of the contract without violating

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<sup>1</sup>Some of us might think that as both  $\bar{y}_0$  and  $\bar{y}_1$  are greater than or equal to 70, the worker will accept the contract, and as  $\bar{y}_1 > \bar{y}_0$ , the worker will be better off by providing effort level  $e = 1$  in comparison to  $e = 0$ . This is not true. As the worker is risk adverse ( $u(w - 10e) = \sqrt{w - 10e}$ ), we must compare the expected utilities under  $e = 0$  and  $e = 1$ , and not expected wages.

<sup>2</sup> $\sqrt{70} = 8.3666$

the incentive-compatibility constraint by offering slightly more insurance (and less incentives). Thus, an efficient contract will provide more insurance and less incentives than  $w = y$ . The firm will find it optimal to choose any such efficient contract that is just generous enough to satisfy the worker's participation constraint.

We finish with a specific example of an intermediate wage contract, offering some insurance and some incentives, that the firm would prefer and the worker would accept. Take  $w = 30 + 0.6y$ . Then, the worker gets a base pay of £30 and 60% of every pound he ends up producing. The expected utilities are

$$\tilde{u}_1 = \frac{3}{4}\sqrt{30 + 60 - 10} + \frac{1}{4}\sqrt{30 + 36 - 10} = 8.579,$$

for  $e = 1$  and

$$\tilde{u}_0 = \frac{1}{4}\sqrt{30 + 60} + \frac{3}{4}\sqrt{30 + 36} = 8.4647$$

for  $e = 0$ . Thus, the worker participates and provides  $e = 1$ . The expected profit is then  $\mathbb{E}[y - 30 - 0.6y|e = 1] = \mathbb{E}[0.4y|e = 1] - 30 = 0.4 \times 90 - 30 = 6$ , so that the firm will prefer this contract over  $w = y$ . Note that the incentive-compatibility constraint is still not binding, so that this contract is not efficient: more insurance can be offered to the worker without hurting incentives. Also, the participation constraint is still not binding so that the firm can increase expected profits by offering a slightly less generous deal to the worker. So, this specific contract is still not efficient nor optimal (from the firm's perspective).

- (e) When information is perfect, the firm can both maximise output (by requiring work effort) and provide full insurance to the worker. We have just seen that, in our example, under asymmetric information it is still efficient to write a contract that induces work effort and therefore maximises output. However, because of moral hazard, this requires that the worker's pay is sufficiently sensitive to work effort. Under asymmetric information, this can only be achieved by making pay output-dependent, in which case it depends not only on effort, but also on chance. Thus, the worker has to bear some risk in order to induce work effort. Because of this trade-off between insurance and incentives, the most efficient contract under asymmetric information is not as efficient as the best contract under perfect information.

3.

- (a) If the worker can choose both the effort level and the wage contract, (s)he will take the one giving her/him the highest expected utility. A normal worker

compares four options (working and shirking under fixed and output-dependent wages), whose expected utility outcomes are given in Table 1:

Table 1: Expected utilities for normal workers

$w \backslash e$	$e = 0$	$e = 1$
$w = 70$	$\sqrt{70} = 8.3666$	$\sqrt{60} = 7.746$
$w = y$	$\bar{u}_0 = 8.3095$	$\bar{u}_1 = 8.8829$

As the highest utility level is reached for  $w = y$  and  $e = 1$ , this would be the contract that a normal worker would choose. As  $\bar{u}_1 > \sqrt{70}$ , the normal worker participation constraint is fulfilled.

The lazy worker will always shirk ( $e = 0$ ). The lazy worker's expected utility is therefore  $\sqrt{70}$  under a fixed wage contract and  $\bar{u}_0 < \sqrt{70}$  under  $w = y$ . Thus, the lazy worker will pick the fixed-wage contract, will shirk and is indifferent to work in the farm or elsewhere in the economy.

- (b) The normal worker is, by 3.a, happy to accept this contract and work. The lazy worker, who always shirks because of enormous effort costs, would derive utility  $\bar{u}_0 < \sqrt{70}$  from this contract and prefers to accept a £70 fixed-wage deal elsewhere.