Structural Duration Analysis

Rede, uitgesproken bij de openbare aanvaarding van het ambt van hoogleraar in de econometrie aan de Universiteit van Tilburg op 9 oktober 2009

door

Jaap H. Abbring
Mijnheer de rector, dames en heren,

Veel van mijn buitenlandse collega’s volgden deze zomer cursussen Nederlands. Dat heeft ze meer Hollandse directheid dan waardering voor onze taal bijgebracht. Ze lieten me weten geen prijs te stellen op drie kwartier Nederlandse voorleeskunst. Ter ere en vermaak van mijn collega’s ga ik daarom verder in het Engels.

Dear colleagues, students, family and friends,

My introduction in Dutch may already have you wondering how long this session will last. Please wonder no more. The time constraints on inaugural lectures are tight, and my proprietary data on their durations are quite clear about what you can expect today (Figure 1).

Figure 1: Inaugural Lecture Duration

The inaugural lecture is delivered during an official session in the Auditorium. Its duration is that of a normal lecture (45 minutes)

— Section 2.1 of the guidelines for inaugural lectures

Less trivial is the question why it took me over a year to schedule this lecture in the first place. In his inaugural lecture a few weeks back, Willem Haemers showed that there is substantial variation in the durations between the appointment and the inauguration of professors in our department (Figure 2). Surely, this variation reflects heterogeneity in our preferences for giving public lectures and in the constraints that we are facing. Knowing these preferences and constraints would help
you understand our choices and the university’s management to optimally control them. You could learn about them from an econometric analysis of Willem’s duration data, based on a proper model of inaugural timing under constraints.

Now, quite frankly, you don’t need to go through all that trouble to see from the raw data that some of us expect low returns from giving a lecture, and that the institutional constraints we are facing are quite loose. In any case, I am glad that you all showed up today, and it will be my pleasure to complete one ongoing inauguration spell in Willem’s data set.

I will do so by lecturing on econometric methods to learn about economic primitives, the preferences and technologies that shape economic behavior and outcomes, from duration data. Such “structural duration analysis” covers a wide range of economic problems.

- For example, in labor economics, unemployment durations have been studied to learn about the perils of long-term unemployment; and the evaluation of policies that target it by offering training, job search and other labor market programs [25, 10].
• In population economics; marriage, divorce, and fertility dynamics have been analyzed in relation to family policies [22].

• In contract economics, insurance contract and claim histories have been analyzed to learn about moral hazard and adverse selection in insurance markets and to inform firms and policy makers about optimal contracts and regulation [9, 11].

• In industrial organization, firm survival has been analyzed in studies of firm growth, entrepreneurial learning, and the efficient allocation of resources [4]. Also, data on the renewal of patents have been studied to determine the value of intellectual property rights, and inform innovation policy [39, 40].

• Finally, in education economics, school leaving decisions can been analyzed to infer the perceived returns to schooling, and to inform education policy [26].

Three approaches to structural duration analysis can broadly be distinguished [1].

• The first of these uses continuous time models in which heterogeneous agents—individuals or firms— are assumed to take discrete decisions at random and discrete— or, Poisson — times. Continuous time sequential job search models are prime examples of such models, and are a key tool in the empirical analysis of labor market dynamics [17]. In them, job offers arrive at Poisson times, and agents decide whether to accept job offers when they have arrived. Similar models have appeared in insurance economics; where agents decide on claiming losses that are incurred at Poisson times [11]. They imply hazard rates for labor market transitions and claims; and they are naturally analyzed with hazard models.

• The second is a discrete time approach in which agents are assumed to solve dynamic discrete decision problems with payoffs determined by persistent pro-
cesses, usually Markov processes. These models have been applied to a wide range of economic problems, including many of the duration examples we have just seen [31].

- The third approach employs continuous time models in which agents make discrete decisions with payoffs determined by Brownian motions or more general persistent processes. These models are central to the options literature in finance, and can be used to empirically analyze a large range of optimal stopping problems of the so-called “real options” type [16, 46]. They typically reduce to models in which durations equal the first times the latent process hits a threshold.

These three approaches have mostly been developed and applied independently in economics, with many fields settling on one class of models or the other. Moreover, the techniques for their analysis differ substantially. It should be stressed, though, that the continuous time and discrete time approaches have much in common substantially. In fact, their distinction is not very sharp in the early literature. On the one hand, empirical job search models have been implemented in both discrete and continuous time. In their early work on the econometrics of job search, Christopher Flinn and James Heckman explicitly pointed out that their continuous time analysis could easily be adapted to discrete time [20]. On the other hand, some of the early applications of discrete time models, such as Ariel Pakes’ study of patent renewal and John Rust’s analysis of bus engine renewal, involved optimal stopping problems similar to those in the continuous time real options literature [39, 44].

In line with the early literature, my lecture today brings together all three approaches. I hope to make two points.

- First, the hazard and hitting-time approaches are substantial complements. That is, they are each appropriate tools for their own classes of economic problems. Hazard models for heterogeneous agents are suitable to the analysis
of job search and insurance problems; hitting-time models should be used when studying problems of the real options type.

- Second, continuous hitting-time models are a useful alternative to similar discrete time models, because they can be analyzed with powerful techniques from the hazard literature. Consequently, important new results can be derived in continuous time, that are not available for the corresponding discrete time case.

In the remainder of this lecture; I will subsequently discuss the hazard, discrete time, and hitting-time approaches; before I conclude with some appropriate reflection and some expression of gratitude for your endurance.
Hazard Models

The financial crisis has brought one problem firmly back into the public spotlight: Unemployment. Unemployment is rising rapidly in many countries. The Centraal Planbureau currently predicts that Dutch unemployment will double from just under 4% of the labor force last year to 8% in 2010 [41] (Figure 3).

With this pessimism about the labor market, old worries about the adverse effects of prolonged unemployment have resurfaced as a major policy issue. Long term unemployment is likely to become more prevalent, because more people will be unemployed and it will take newly unemployed longer to find jobs. This could be a problem if unemployment changes a worker’s skills or preferences.[25, 24]. For example, an unemployed worker who lossees skills when unemployed, may find it increasingly hard to attract good job offers. In fact, the temporary Dutch part-time unemployment insurance scheme is in part motivated by worries about such loss of skills.

A key piece of evidence on the perils of long term unemployment is the fact that long term unemployed move into jobs at a lower rate than newly unemployed.
Figure 4: Monthly Exit Probabilities from Unemployment Insurance (WW) and Welfare (WWB) to Employment by Elapsed WW and WWB Durations; the Netherlands, 1999–2005

Figure 4 plots the monthly exit probabilities from Dutch unemployment insurance (WW) and welfare (WWB) by time spent in those programs. In both programs, job transition rates fall dramatically with time spent unemployed. For example, workers who have been on unemployment insurance for more than three years are five times less likely to exit to employment than those who have just entered the program.

Similar patterns are observed for other time periods and countries. There are two interpretations:

- First, there may be state dependence at the level of the individual. That is, unemployment may breed unemployment by reducing the individual’s skills or motivation. Then, each individual worker experiences a decrease in reemployment opportunities with the time spent unemployed (Figure 5).

- Second, workers may differ in their reemployment opportunities to begin with, because of heterogeneity in their preferences and skills. Because unemployed with low reemployment chances are more likely to survive in unemployment,
the observed average reemployment rate falls with duration even if individual reemployment rates are not affected by continued unemployment (Figure 6).

If we could observe all individual characteristics that determine variation in reemployment probabilities across unemployed workers, then we could plot reemployment probabilities for homogeneous groups of unemployed, for various values of the characteristics. Any duration dependence within groups of unemployed would reflect genuine state dependence; any variation in reemployment rates across groups would reflect heterogeneity. With many and continuous characteristics, it may not be fea-
sible to stratify the data into homogeneous groups. Then, Cox proportional hazards analysis, available in any decent statistical package, would offer a straightforward way to measure duration dependence for given individual characteristics [14].

Usually, these methods do not uncover true duration dependence, because micro data do not provide information on all relevant individual characteristics. Clearly, such unobserved heterogeneity cannot be distinguished from duration dependence without further structural assumptions [34]. This has inspired an extension of the Cox proportional hazards model with unobserved heterogeneity, by Tony Lancaster in the late 1970s [33]. This extension, the mixed proportional hazards model, has developed into the most popular model for econometric duration analysis, with implementations in various statistical packages. The model’s key proportionality assumption is that the ratio of any two agents’ reemployment hazards is constant over time. For example, Figure 7 plots proportional hazard paths for two different agents. The hazards differ because of observed and unobserved heterogeneity, but that difference is tightly structured: The agent with the highest path is twice as likely to move into employment at all durations.

In a range of papers, Chris Elbers, Geert Ridder, James Heckman and others showed that the proportionality restriction, with some further assumptions, allows
us to determine the individual-level duration dependence and the effects of observed and unobserved heterogeneity from data on durations and covariates \[19, 27, 42, 1\]. This leaves us with two questions.

- First, what exactly do we learn from knowing duration dependence and heterogeneity in reemployment hazards?
- Second, how much faith should we put into results derived on, what seems to be, a fairly arbitrary proportionality assumption?

Economic theory facilitates disciplined reflection on these questions. In particular, labor market dynamics are often studied with job search theory. In a basic sequential job search model, agents receive job offers at some Poisson rate. Job offers are simply wage offers, drawn from some wage offer distribution. Once a job offer arrives, agents decide between accepting the offer or continuing search without the opportunity to recall the offer later. The resulting hazard rate for the transition from unemployment to employment is the product of the job offer arrival rate and the probability that a job offer is accepted. In the simplest such setting, the job offer arrival rate and the job offer distribution are taken to be primitives, determined outside the model, which may vary between agents and over time. The model predicts that the agent accepts any job that offers a wage above some agent-specific and time-dependent threshold, the so called “reservation wage”. The job acceptance probability then is the probability that the job offer is better than the reservation wage.

Now, consider the hypothesis that, because of loss of skills, all agents face the same declining job offer arrival rate as they continue to be unemployed (Figure 8). Furthermore, suppose that agents differ in their love of leisure. In particular, consider two types of agents: “Workers”, who derive low payoffs from being unemployed; and “shirkers”, who receive high utility from unemployment. Here, shirkers may appreciate unemployment more because they receive higher unemployment in-
Figure 8: Sequential Job Search by Myopic Agents

The figure also plots the job offer acceptance probabilities indirectly chosen by the two types of agents. Here, I have assumed that the agents are “myopic”, in the sense that they do not care about the future at all. That is, they seize the day by accepting any job offer that offers a wage that beats their immediate payoff from being idle, irrespective of their further opportunities in the labor market if they would forego the job offer. Then, workers and shirkers choose constant but different reservation wages, with workers accepting a larger share of job offers than shirkers. The corresponding reemployment hazard rates are the product of the common and time-varying arrival rate and the heterogeneous and time-constant acceptance probabilities.

Clearly, in this case, the hazards of both types of workers are proportional, and the proportional hazards assumption is justified. This implies that we can invoke the results for the mixed proportional hazards model to disentangle duration dependence and heterogeneity in the reemployment rates. In turn, this allows us to assess the decline in the job offer arrival rate relative to its level for newly unemployed and test the loss-of-skills hypothesis. Note however that, even in this simple case, we cannot say much about the levels of the job offer arrival rate and the acceptance...
probability. In their seminal analysis of econometric job search models, Christopher Flinn and James Heckman showed that it is impossible to tell from unemployment duration and accepted wage data whether reemployment rates are low because offer rates or low or because acceptance probabilities are low, unless the class of job offer distributions is restricted to satisfy a so called “recoverability” condition [20]. So, in this simple myopic case, we can empirically assess a key structural implication of loss of skills, duration dependence of the reemployment rate, but we cannot determine the full model structure.

If unemployed are forward looking, and do not only consider the current payoffs from a job and unemployment, but also their future job search opportunities, the analysis becomes more complicated [47]. In this case, the unemployed will take note of the decline in the rate at which jobs are offered, and will be increasingly willing to accept job offers. Figure 9 plots an example under the simplifying assumption that agents, when considering their future opportunities, incorrectly assume that the job offer arrival rate will remain constant at its current level. A similar plot could be drawn under the alternative assumption that agents perfectly foresee their future job offer arrival rates. The figure displays a robust prediction of such models: The
shares of job offers accepted by workers and shirkers increases as long as the offer arrival rate decreases, and they increase nonproportionally. As a consequence, the implied reemployment rates for workers and shirkers are not proportional either: At higher durations, the gap between their reemployment rates is relative wide. This implies that the mixed proportional hazards model cannot be used to learn about duration dependence and heterogeneity in this case.

In sum, a mixed proportional hazards analysis may uncover individual level effects of continued unemployment on reemployment rates and heterogeneity in those rates. However, if agents look forward when choosing reservation wages, the implied reemployment rates are usually not proportional across agents.

This conclusion extends beyond job search and unemployment durations, to other applications with economic behavior driven by Poisson shocks. One such application is the empirical analysis of moral hazard by analyzing insurance claim histories. In papers with Pierre-André Chiappori, James Heckman, Jean Pinquet, and Tibor Zavadil, we have studied moral hazard in car insurance data from the files of French and Dutch insurance companies [8, 9, 11]. Absent fraud, insurance claims are the result of discrete claiming choices that follow random losses incurred at random times. Moral hazard entails that the agents—the insurees—have better information about their risk behavior than their insurance companies, and are less careful to avoid losses and claims if they have better coverage. So, if there is moral hazard, the rate at which losses occur, the sizes of these losses, and the likelihood losses are claimed may all be larger if the “incentives” to avoid claims are smaller; that is, if the deductible, copayment, or bonus-malus punishment are smaller. If, instead, there is no moral hazard, then losses and claims do not respond to variation in such incentives.

This suggests that we can empirically test for moral hazard by checking whether losses and claims vary with the incentives agents face to avoid claims. For example,
Table 1: Bonus-Malus (BM) Scheme Dutch Car Insurance

<table>
<thead>
<tr>
<th>Present BM class</th>
<th>Premium paid</th>
<th>Future BM class after a contract year with no claim</th>
<th>1 claim</th>
<th>2 claims</th>
<th>3 or more claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>25%</td>
<td>20</td>
<td>14</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>25%</td>
<td>20</td>
<td>13</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>25%</td>
<td>19</td>
<td>12</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>25%</td>
<td>18</td>
<td>11</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>25%</td>
<td>17</td>
<td>10</td>
<td>6</td>
<td>1</td>
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<tr>
<td>15</td>
<td>25%</td>
<td>16</td>
<td>9</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>25%</td>
<td>15</td>
<td>8</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>30%</td>
<td>14</td>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>35%</td>
<td>13</td>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>37.5%</td>
<td>12</td>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>40%</td>
<td>11</td>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>45%</td>
<td>10</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>50%</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>55%</td>
<td>8</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>60%</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>70%</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>80%</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>90%</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>100%</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>120%</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

if we have data on contracts and claims, we could check whether agents with low deductible contracts have more or larger claims. We should be careful though, because any such relation may reflect selection on unobservable risk, factors rather than the moral hazard effects of incentives. For example, if some agents are inherently more risky than others, this information is private, and insurance companies offer a choice from a menu of contracts with various levels of coverage; then high risk agents will choose low deductible contracts [43]. Then, we will observe that agents covered by low deductible contracts, which provide relative small incentives to avoid claims, have relatively many or large claims, even if there is no moral hazard.

Therefore, in a recent paper with Pierre-André and Tibor, rather than focusing on variation in incentives across agents, we exploit variation in incentives within an agent’s relationship with an insurance company. We use data on individual car insurance claim and contract histories from a Dutch insurance company. The agents face the bonus-malus scheme in Table 1, which is fairly standard in the
Netherlands and should be familiar to most of you who drive cars. New drivers start in bonus-malus class two and pay 100% of some base premium. Then, each year the bonus-malus class, and the corresponding discount, are updated depending on the number of claims filed in the year. If no claim is filed, the agent moves to a higher class, and receives a higher discount on the base premium. With each claim, the agent moves down, and receives a lower discount. It should be clear from this table that incentives change discretely whenever a claim is incurred; and that the changes differ between bonus-malus classes. For example, with her first claim, an agent in class five moves from paying 60% of the base premium next year to paying twice that amount; but she does not pay a further price for filing additional claims. In contrast, an agent in class 20 would still be in class 14, with the same 75% discount, after one claim; but would pay twice as much premium after a second claim, and nearly five times as much after a third claim.

Figure 10 further explores this variation in incentives by plotting the expected discounted increase in future premium payments with each claim filed, for an agent with average risk. This number reflects the fact that a claim not only increases the premium next year, but also in the years after that. It is simply the current capital loss because of the claim. The solid line plots this capital loss for the first claim filed in a contract year, by bonus-malus class. Indeed, agents in low classes incur a substantial loss of up to three times the annual base premium in class five. Even agents in class 20 face a capital loss from a first claim, even though they would still have the same discount next period if they do not incur another claim. The reason for this capital loss is that the first claim puts agents in class 20 at higher risk of loosing their discount in the future, because of another claim. The loss is small, because this risk is small.

The capital losses from a first claim can be contrasted with the additional losses that would be incurred with a second and third claim in the year. These are absent for the lowest classes, because agents in these classes have already reached the bot-
Figure 10: How Many Times the Annual Base Premium Do You Lose with Each Claim?

Source: Abbring, Chiappori and Zavadil [11]. $\Delta V(1, K, N)$ denotes the decrease in expected discounted utility when a claim is filed at the end of the contract year, just before the premium is updated; by an agent in bonus-malus class $K$, who has already filed $N - 1$ claims in that same contract year and faces a sample average risk level. The bottom graphs for each $N$ correspond to the risk neutral case, and plot the expected discounted premium increases in multiples of the annual base premium (that is, before BM discounts). The higher graphs correspond to increasing levels of constant absolute risk aversion.

tom of the bonus-malus system with their first claim. In contrast, the capital losses are relatively large in the higher classes, where premium increases only start kicking in after the second claim. The capital loss can be over 3.5 times the annual base premium in the highest classes. Because agents in those classes pay only 25% of the base premium, this amounts to a punishment of over 14 years of insurance premium payments. In fact, if you are anything like the Dutch population, you will be in one of the top classes, facing such steep prices for claiming.

So, the incentives to avoid claims are very large in some states. Moreover, they jump with each claim filed. In the lower classes, incentives drop to zero after the
first claim; in the highest classes, they instead increase. This implies that, under moral hazard, claim rates increase after the first claim in the lowest classes. In the highest classes they decrease. This suggests a test for moral hazard based on the dependence of claim rates on the previous occurrence of claims. This test should again take into account that such true state dependence may be confounded with selection on unobserved risk factors.

We formalize this by developing a model of the claim behavior of a rational and forward-looking agent under moral hazard. Random losses arrive at some Poisson rate. The agent can exert costly effort to bring the loss rate down. If a loss arrives, he has to decide whether to claim it, or pay for it out of pocket. Because the bonus-malus cost of claiming is independent from the loss amount, but the benefits of claiming increase with it, the agent will claim any amount above some threshold. Like the reemployment rate in the job search model, the claim rate is the loss rate times the probability that a loss is claimed. The model predicts that both go down if the incentives, roughly as measured by the expected discounted premium increase, rise. Under moral hazard, the claim hazard depends in a complicated, nonproportional way on individual risk factors, time, and the claim history. A proportional hazards analysis of the state dependence and heterogeneity in the claim rate would not quantify the true state dependence due to moral hazard. Instead, we develop a score test for moral hazard in the context of the structural model. We find strong evidence of moral hazard, and leave its quantitative assessment for future work.

More generally, one may be interested in analyzing the relation between multiple discrete events. The statistician John Freund presented an early example, a bivariate duration model in which the realization of one event may shift the rate at which another event occurs [21]. He motivated his model with the example of a two-engine plane. A two-engine plane can typically fly with only the remaining engine after one engine fails. However, the added stress on the remaining engine may increase
the risk of that engine failing as well. This would cause a positive association between the two engines of a plane failing and can be tested with engine failure data. A complication is again that unobserved heterogeneity may also cause such an association. For example, some planes may be better maintained than others, so that a poorly maintained engine is likely to be paired with another poorly maintained engine.

With Gerard van den Berg, I have extended Freund’s framework with unobserved heterogeneity and covariates that may capture such selection effects, and give conditions under which state dependence and selection can be distinguished [7]. In work with Gerard and Jan van Ours, we apply this model to the analysis of the moral hazard effects of unemployment insurance sanctions—punitive benefits reductions—on the reemployment hazard [10]. We seek to distinguish these effects from the effects of unobserved labor market characteristics that affect both reemployment and sanction rates. We again specify a sequential search model with random benefits reductions to motivate this analysis, and much of the earlier concerns about the theoretical justification of the econometric model framework carry over to the analysis here.

The timing-of-events framework has been applied to many other problems. For example, Pieter Gautier, Michael Svarer and Coen Teulings observed that urban couples are far more likely to divorce than rural ones [22]. One hypothesis is that
urban marriages are less stable because urban marriage markets offer many more opportunities for search on the match. An alternative hypothesis is that unstable marriages sort into cities, for example because they value the option to search for partners if the need would arise. Pieter and his coauthors use data on divorces and residential moves between urban and rural areas, and the timing-of-events framework, to distinguish between these hypotheses. They find that sorting, not state dependence, explains the data. I guess that the fact that we still live in Amsterdam reflects poorly on my common-law marriage with Barbara, but does not hurt it.
Discrete Time Models

It seems that most of you are still here. I am worried though that some of you will leave this room before I am done. For all I know, you are continuously evaluating the possibility of exiting against hanging on to the option of doing so later, based on your evolving assessment of how good my talk will be and what cost you will incur for your public departure. I am not so much worried about sudden incidents that may make you leave, but rather about you slowly but surely developing a taste for the drinks and snacks outside. So, I cannot evaluate this process using the methods based on the Poisson arrival of shocks that we have discussed so far. Rather, I should model your exit decision as being driven by your evolving appreciation for being here or outside.

Fortunately, a wide range of economic problems fit this same mould. I will first briefly discuss a large discrete time literature that addresses these problems, before I return to related, recent developments in continuous time duration analysis.

A seminal contribution to structural dynamic discrete choice analysis is John Rust’s study of Harold Zurcher [44]. Mr. Zurcher was in charge of maintenance of a
fleet of Wisconsin buses (Figure 13). In particular, he was responsible for deciding when to replace the engine of each bus in his fleet. Key inputs into these decisions were the odometer readings on the buses: An engine that had traveled further commanded higher regular maintenance costs, and was a more attractive candidate for replacement. John Rust had data on Mr. Zurcher’s decisions and the odometer reading on each bus at each point in time, and used these data to estimate a so-called “regenerative optimal stopping model” of Harold Zurcher. He assumed that Mr. Zurcher solved a dynamic optimization problem given the data at hand, the number of miles each engine had traveled and some further variables known to Mr. Zurcher but not to John Rust. Rust’s analysis relies on the key simplifying assumption that Mr. Zurcher cannot predict future maintenance costs any better than John Rust. That is, all information about future costs is embodied in the history of odometer readings observed by both Mr. Zurcher and John Rust; the other variables privately observed by Mr. Zurcher may affect his decisions through current maintenance costs, but do not help him predict future payoffs. This “conditional independence” assumption rules out dynamic selection on unobservables of the type seen in the first
part of this lecture. So, the odometer readings and engine renewal history of a bus carry no information on the future values of the cost factors privately observed by Mr. Zurcher.

Even under the conditional independence assumption, as with job search models, panel data on discrete choices and observed state variables do not fully determine all the primitives of a model like this [37, 45]. However, recent research has made precise which assumptions or external information are needed to fully determine all primitive parameters [36].

Much empirical and econometric research relies on Rust’s conditional-independence assumption, which greatly simplifies the solution of the agent’s dynamic decision problem and its subsequent empirical implementation. However, in many economic applications, the assumption that the econometrician is as able to predict an agent’s future as the agent herself is not justified.

The simplest way to relax this assumption is to allow for finite unobserved heterogeneity. In their analyses of schooling and labor market behavior; Zvi Eckstein, Kenneth Wolpin, Michael Keane, and others suppose that there are a finite number of types in their data set, each with their own preferences and abilities [30, 18]. Each agent knows her type, but types are not observed by the econometrician. Because agents’ types affect their decisions, data on choice and observed covariate histories are typically informative on the agent’s types. For example, if a kid is observed to drop out of school early, he is more likely to be of a type that perceives schooling to have a low return. So, there is nontrivial dynamic selection on unobservables, and the relation between observed choices and observed state variables confounds true state dependence and selection effects.

Only recently, progress has been made in answering the “identification question” whether these effects can in principle be uniquely determined from dynamic choice and covariate data. Hiroyuki Kasahara and Katsumi Shimotsu provide conditions
for the identifiability of dynamic discrete choice models with finite heterogeneity, building on the classical literature on finite mixtures in statistics [29].

Some applications call for generalizations with unobservables that may vary over time. An early example is Ariel Pakes and Margaret Simpson’s analysis of the returns to patent protection [39, 40]. They note that patent holders need to pay an annual renewal fee to keep their patents in force. This implies that data on their patent renewal behavior are informative on the value they attach to their patents. This is useful, because patents are usually not marketed, so that we cannot infer their values from market prices. Pakes and Simpson specify a model in which patent holders make optimal dynamic renewal choices, given a renewal fee schedule and a general Markov process for the returns to their patents. They show that the patent returns process can be uniquely determined from data on patent renewal choices, provided that sufficient variation in renewal fees across patents and over time is observed. Intuitively, patents will only renewed if they are worth more than the renewal fee, so that information on the shares of patents renewed across different renewal fees is informative on the distribution of the values from holding patents.

Flavio Cunha, James Heckman, and Salvador Navarro generalize this framework in the context of schooling choice and returns to schooling [15, 26, 5, 6]. They allow for very general processes for the unobservables and use earnings data to provide direct information on the returns to schooling. As in Pakes and Simpson’s work, they require sufficient variation in schooling costs with observed covariates to determine their model primitives from data on schooling choices and earnings. Again, the agents’ dynamic schooling choices under various cost regimes are informative about the returns to schooling as perceived by the agents.

In a similar vein, with Jeffrey Campbell, I have developed a model of firm growth, learning, and survival [4]. Our main objective is to characterize the accumulation of information to entrepreneurs about their firms’ profitability, and to assess the en-
trepreneurs’ effectiveness in selecting profitable firms for survival. As in the schooling example, we adjoin data on firm performance to data on firm survival. However, we cannot rely on observed variation with external covariates, as none are available. Instead, we settle for a more tightly structured process for the unobserved profit determinants.

It should be clear from these examples that a wide range of economic problems can be studied with economic models based on persistent state processes. I will now present a continuous time version of such a model that is closely aligned with important continuous time models in economic theory, and for which novel econometric results can be derived.
Hitting-Time Models

Continuous time models driven by Brownian motion, or more general persistent processes, are gaining popularity in statistics [35]. They are central to the options literature in finance, and can be used to empirically analyze optimal stopping problems of the real options type, discussed in Avinash Dixit and Robert Pindyck’s seminal monograph and Nancy Stokey’s recent book [16, 46]. They often reduce to models in which durations equal the first times the latent process hits a time-invariant and heterogeneous threshold.

One important example is Robert McDonald and Daniel Siegel’s analysis of investment timing [38]. In their model, agents are endowed with an option to invest in a project, at a time of their choice. Investment incurs a given cost; in return, the agent receives the project’s value at the time of the investment. The log of this value follows a Brownian motion. The agent maximizes her expected discounted payoffs by investing when the project’s value hits a time-invariant threshold. Figure
Figure 15: Hitting Time Hazard Rates

14 plots such first hitting times for two realizations of the log project value. Primitive heterogeneity; such as variation in initial project values, investment costs, and discount rates across agents; induces heterogeneity in the threshold.

Another example is a search-matching model of job tenure with heterogeneous search frictions and no search on the job. In more common terms, consider how long workers stay in a given job; assuming that finding another job takes time, some need more time than others, and only unemployed can search. All jobs initially offer the same wage, but are subsequently hit by job-specific persistent shocks. Then, under some assumptions on these shocks, for example that log wages follow a Brownian motion, workers leave their job when their wage falls below a fixed threshold. Workers who can very easily find another job will not tolerate any wage loss, and have a threshold just below the initial wage. Workers who face substantial job search frictions will however accept wages below their initial wage, as long as the possibility of future wage increases sufficiently compensates for the current low wage. So, heterogeneity in search frictions generates heterogeneity in the job exit threshold, and job durations equal the first times log wages hit a heterogeneous threshold.

This implication is common to many heterogeneous optimal stopping models driven by Brownian motion, or more general processes with independent and sta-
tionary increments [16, 46, 32, 13]. This motivates the use of a mixed hitting-time model, which specifies durations as the first times some latent process hits a threshold that may depend both on observed covariates and unobserved heterogeneity [2]. Mixed hitting-time models imply hazard rates, but they do not produce them directly from primitives like job search and insurance models do. In fact, the hazard rate paths implied by different thresholds are usually far from proportional (Figure 15). This implies that a proportional hazards analysis of data generated from, for example, the search-matching model of job tenure would not correctly disentangle the effects of job-specific wage dynamics and heterogeneity in search frictions on job tenure.

In a recent paper, I show that the analysis of the mixed hitting-time model is mathematically close to that of the mixed proportional hazards model, even though it is substantially distinct from it. In particular, many of the identification results for the mixed proportional hazards model can be translated to the mixed hitting-time model. This gives weak conditions under which the latent process and the threshold heterogeneity can be uniquely determined from duration and covariate data. This way, the continuous time framework provides a useful complement to similar discrete time models. Possible applications in other fields than labor economics include marriage and divorce, firm entry and exit, and credit default.
Conclusion

It is time to converge on some conclusions.

• First, hazard models are natural for the analysis of job search and insurance problems. Sometimes, mixed proportional hazards models are consistent with job search theory, and can be used to distinguish state dependence and unobserved heterogeneity. Often, they are inconsistent with economic theory, and they would not produce structurally meaningful empirical results.

• Mixed hitting-time models are natural for the analysis of optimal stopping problems driven by Brownian motion or more general persistent processes, which do not lead to proportional hazards models. They complement similar discrete time models with a novel identification analysis made possible by their continuous time specification. This analysis is closely related to that of the mixed proportional hazards model. So, even in cases where the mixed proportional hazards model cannot be applied, results derived for the proportional hazards framework can be put to good use.

• Finally, the literature’s conclusion that the mixed proportional hazards model can only be used for descriptive analysis, not for structural analysis [17] is both too strong and too weak. On the one hand, mixed proportional hazard models may be consistent with economic theory and yield structurally useful results. On the other hand, in applications where the mixed proportional hazards model cannot be anchored in economic theory, there is little point in using it at all. The Cox proportional hazards model, without unobserved heterogeneity, is a very versatile and convenient tool for descriptive duration analysis [14]. Extending Cox’s framework with unobserved heterogeneity greatly complicates its analysis, without obvious descriptive benefits.

I expect three topics to dominate my research agenda in the near future.
• First, I will extend the mixed hitting-time model to include time-varying co-

variates as noisy measurements of the latent process, as in some of the discrete
time models discussed.

• Second, I will investigate whether the mixed hitting-time model can be ex-
tended to analyze optimal stopping games. Bo Honoré and Aureo de Paula
recently applied hazard model results to what is, essentially, a static timing
game [28]. The hitting-time framework may allow the analysis of a truly dy-
namic game.

• Third, with Jeffrey Campbell and Nan Yang, I have been and will be working
on the computational and empirical analysis of discrete time discrete games,
in the context of the analysis of market structure dynamics [3]. This research
builds closely on the results for discrete time discrete choice models discussed
today [12].

This brings us to the end of my lecture. I greatly appreciate that you all showed
up and stayed around. I took a while to schedule this lecture, but only to facilitate an
honest and well-informed assessment of academic life in Tilburg. I can now say, with
confidence, that Tilburg University offers wonderful colleagues in the Econometrics
department and elsewhere in the School, kind provision of Nespresso technology by
the Economics department, and plenty of room for research and teaching initiatives.

There is one such initiative, the Structural Econometrics Group, that I would like
to explicitly mention here. This informal and open group brings together faculty
and students from the Econometrics, Marketing, Economics and Finance depart-
ments who are interested in structural econometrics and computational economics;
and in their empirical applications to industrial organization, marketing, household
economics, and labor economics. It focuses on the dissemination of information on
relevant activities at Tilburg University (Figure 16); and the organization of dedi-
cated seminars, incoming visits, and courses. I greatly appreciate the opportunity
to meet with colleagues and students through this group, and I invite everyone interested, and in particular also research students, to join.

Before saying a few words in Dutch to my family (Figure 17), let me conclude by gratefully thanking the management of the University, the Faculty and the Econometrics department for hiring and supporting me. I will work hard to make this a mutually beneficial arrangement.

Tot slot moet u weten dat mijn dochter Hanne vandaag zes jaar is geworden. Hanne, ik vind het heel lief dat je op je verjaardag ook naar mijn feestje wilde komen! Lieve Bram, je bent nog niet eens vier jaar en toch heb je naar mijn praatje geluisterd; ik ben trots op je. Lieve Barbara, je ambt in het openbaar aanvaarden, dat is eigenlijk gewoon trouwen met je baan. Ik ben nu al twee keer getrouwd met mijn baan. Voor het volgende feestje moeten we echt iets anders verzinnen.

Ik heb gezegd.
Figure 17: Hanne (October 2003), Barbara, Bram (November 2005) en Jaap
References


